

Lesson 11. Multivariable Functions

1 This lesson...

- How do functions of many variables work? What do they look like graphically?

2 Functions of 2 variables

- A **function f of 2 variables**
 - takes an ordered pair (x, y) of real numbers as input
 - outputs a unique real number $f(x, y)$
- The **domain D** of f is the set of allowable inputs to f
 - If f is given by a formula and its domain is not explicitly specified, then the domain of f is the set of all (x, y) for which the formula is well-defined
- The **range** of f is the set of values that f takes on

Example 1. Let $f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1}$.

- a. What is $f(3, 2)$?
- b. What is the domain of f ?

Example 2. In 1928, using economic data published by the government, Charles Cobb and Paul Douglas modeled production output $P(L, K)$ as a function of the amount of labor involved L and the amount of capital invested K :

$$P(L, K) = 1.01L^{0.75}K^{0.25}$$

This function (in a more general form) is known as the *Cobb-Douglas production function*.

Find $P(120, 20)$. In words, what does $P(120, 20)$ mean?

- Functions are not always represented by explicit formulas, as the next example shows

Example 3. The *wind-chill index* $W(T, v)$ is a subjective temperature that is a function of the actual temperature T (in $^{\circ}\text{C}$) and wind speed v (in km/h), as given by the table below:

		Wind speed (km/h)											
		v	5	10	15	20	25	30	40	50	60	70	80
Actual temperature ($^{\circ}\text{C}$)	T	5	4	3	2	1	1	0	-1	-1	-2	-2	-3
	0	-2	-3	-4	-5	-6	-6	-7	-8	-9	-9	-10	
	-5	-7	-9	-11	-12	-12	-13	-14	-15	-16	-16	-17	
	-10	-13	-15	-17	-18	-19	-20	-21	-22	-23	-23	-24	
	-15	-19	-21	-23	-24	-25	-26	-27	-29	-30	-30	-31	
	-20	-24	-27	-29	-30	-32	-33	-34	-35	-36	-37	-38	
	-25	-30	-33	-35	-37	-38	-39	-41	-42	-43	-44	-45	
	-30	-36	-39	-41	-43	-44	-46	-48	-49	-50	-51	-52	
	-35	-41	-45	-48	-49	-51	-52	-54	-56	-57	-58	-60	
	-40	-47	-51	-54	-56	-57	-59	-61	-63	-64	-65	-67	

- Find $W(-15, 40)$. In words, what does $W(-15, 40)$ mean?
- Define the function $h(T) = W(T, 40)$. Describe the behavior of h .

3 Functions of n variables

- A function f of n variables

- takes an ordered tuple (x_1, \dots, x_n) of real numbers as input
- outputs a unique real number $f(x_1, \dots, x_n)$

Example 4. Anteater-Bugs produces n types of beers. It costs c_i to produce one bottle of type i beer ($i = 1, \dots, n$). Let $C(x_1, x_2, \dots, x_n)$ be the total cost of producing x_1 bottles of type 1 beer, x_2 bottles of type 2 beer, \dots , and x_n bottles of type n beer.

Write a formula for $C(x_1, x_2, \dots, x_n)$.

4 Graphs of functions in 2 variables

- Let f be a function of 2 variables with domain D
- The **graph** of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D

Example 5. Sketch the graph of $f(x, y) = x^2 + 2y^2 + 1$ by first drawing its traces for $z = -1, 0, 1, 2, 3, 4$.



